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AN ANALYTICAL INVESTIGATION OF A SIMPLIFIED THRUST-VECTOR ORIENTATION TECHNIQUE FOR ESTABLISHING LUNAR ORBITS

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SUMMARY

16565

An analytical study has been made of a simplified guidance technique designed to place a spacecraft in a close orbit around the moon. The motion of the spacecraft during lunar orbit establishment was assumed to be planar. The technique consists in maintaining a constant angle between the thrust axis of the spacecraft and the line of sight to the lunar horizon during the braking maneuver while the orbit is being established. It was found that near-circular parking orbits which are closely confined to the pericynthion altitude of the approach trajectory (80 international nautical miles or 148 160 m) can be established very efficiently by the use of the simplified guidance technique. The simplicity introduced by this technique appears to make it useful for manual control.

Author

INTRODUCTION

Considerable attention has been given at the Langley Research Center in recent years to the development of simplified guidance techniques for various phases of the lunar mission. (See refs. 1 to 8 and references of ref. 1.) These techniques require a minimum of instrumentation and could be used by a pilot to monitor the functioning of an automatic guidance system or as backup control modes to increase the probability of successfully completing an assigned mission.

One phase of the lunar mission which has not received sufficient attention in the area of simplified guidance techniques is the transfer from the earth-moon trajectory to a selenocentric parking orbit. A simplified technique for establishing a selenocentric parking orbit was considered in reference 5. Thrust-vector control was provided by maintaining constant thrust angles between the thrust axis and the line of sight to the horizon during the braking maneuver. The technique required that two constant angles be employed, each maintained for a prescribed time interval.

The purpose of this investigation is to examine the problem of establishing a selenocentric parking orbit by using a single constant-thrust-attitude angle with respect to the lunar horizon during the entire braking maneuver. Particular attention is given to the efficiency of the simplified control technique under a variety of conditions. In addition, a method of modifying the control logic is devised to compensate for initial velocity errors representative of hyperbolic approach trajectories different from the nominal trajectory. Motion of the spacecraft during the mission was assumed to be planar.

SYMBOLS

The units used for the physical quantities defined in this paper are given both in U.S. Customary Units and in the International System of Units (SI). Where distances are expressed in nautical miles or in feet, the international nautical mile and the international foot, respectively, are intended. Factors relating these two systems of units are given in reference 10.

F	thrust, pounds (newtons)
g_e	gravitational acceleration at surface of earth, 32.2 feet per second per second (9.814 meters per second per second)
g_m	gravitational acceleration at surface of moon, 5.32 feet per second per second (1.6215 meters per second per second)
h	altitude above lunar surface, feet or nautical miles (meters)
I_{sp}	specific impulse, seconds
K	angle between thrust vector and line of sight to lunar horizon, degrees (fig. 2)
m	mass, slugs (kilograms)
r	radial distance from center of moon, feet (meters)
r_m	radius of moon, 5 702 000 feet (1 737 970 meters)
t	time, seconds
ΔV	characteristic velocity, $I_{sp}g_e \log_e \frac{m_o}{m_o + \dot{m}t}$, feet per second (meters per second)

W weight, mg_e, pounds (newtons)

α thrust attitude with respect to local horizontal, degrees (fig. 2)

θ angular travel over lunar surface, degrees

ϕ true anomaly of approach trajectory, degrees

Subscripts:

o initial conditions of braking maneuver

A apocynthion conditions

P pericynthion conditions

Dots over symbols indicate derivatives with respect to time. A Δ preceding a parameter indicates a change in that parameter from a nominal value.

STATEMENT OF PROBLEM

The phase of the lunar mission considered in this study is shown in figure 1. The spacecraft is approaching the vicinity of the moon on a nominal approach trajectory. Relative to the moon, the vehicle has hyperbolic velocity and thus will not be captured by the lunar gravitational field. The problem with which this study is concerned is the transfer of the spacecraft from this hyperbolic orbit to a nearly circular selenocentric orbit of a specified radius. In order to make the desired transfer, a retrograde maneuver must be made and, since fuel consumption is of primary importance, an efficient transfer is desired. To this end, a minimum-fuel trajectory program was used in conjunction with the constant thrust angle program so that (a) an efficient thrust angle with respect to the lunar horizon could be chosen and (b) the efficiency of the simplified control technique employed could be evaluated.

Equations of Motion

The equations of motion used in the analysis of the simplified control technique were

$$\ddot{r} - r\dot{\theta}^2 = \frac{F}{m} \sin \alpha - g_m \left(\frac{r_m}{r}\right)^2 \tag{1}$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = -\frac{F}{m}\cos\alpha \tag{2}$$

where

$$m = m_o + \int \dot{m}\, dt$$

and

$$-\dot{m} = \frac{F}{g_e I_{sp}}$$

These equations describe planar movement of a point mass near a spherical, homogeneous nonrotating moon. The angle α describes the orientation of the thrust vector with respect to the local horizontal (fig. 2). A constant-thrust engine was assumed to perform the braking maneuver. The initial thrust-weight ratio was assumed to be 0.262 (i.e., $F/W_o = 0.262$), and the specific impulse of the fuel was assumed to be 313 seconds. The equations of motion were solved on a digital computer.

Nominal Approach Trajectory

The earth-moon trajectory selected as a nominal trajectory for this study is representative of those presently being considered for the lunar mission. The nominal approach trajectory chosen has an earth-moon transfer time of 70.5 hours. A portion of the true anomaly history with respect to the moon is shown in figure 3 where altitude h, radial velocity $\dot{r}$, transverse velocity $r\dot{\theta}$, and time from pericynthion t are plotted against the true anomaly. The minimum altitude of the nominal approach trajectory is 485 000 ft (about 80 n. mi. or about 148 160 m) with a corresponding transverse velocity of 8308 ft/sec (2532 m/s). Initial conditions of the braking trajectories considered in the study correspond to a true anomaly range extending from -20° to -5°.

Optimum Braking Trajectories

A digital-computer program which uses a steepest-descent optimization procedure (ref. 9) was used to compute fuel-optimum braking trajectories from different locations along the nominal approach trajectory. The initial conditions of the braking trajectories corresponded to selected locations on the nominal approach trajectory, and the terminal conditions corresponded to an 80-nautical-mile-altitude circular parking orbit.

Previous studies (see refs. 2 to 8) have shown that a simple control parameter for efficiently performing various phases of the lunar mission is the angle between the vehicle thrust vector and the line of sight to a convenient visual reference. The simple control parameter examined in this study is the angle between the vehicle thrust vector and the line of sight to the lunar horizon. This angle, designated K in figure 2, can be expressed as

$$K = \alpha + \cos^{-1} \frac{r_m}{r_m + h} \tag{3}$$

By the use of equation (3), the K variation during a fuel-optimum braking maneuver can be determined. Figure 4 shows this variation for three fuel-optimum braking maneuvers initiated at representative altitudes of approximately 721 000, 586 000, and 500 000 feet (219 761, 178 613, and 152 400 m). K is shown plotted against thrust time. With the exception of the 586 000-foot-altitude case, K varies only a few degrees during the fuel-optimum maneuvers, and hence could be represented by a constant angle. To facilitate the determination of the constant angle to be used, an integrated average of K was computed for a number of optimum trajectories. By examining values of K near the integrated average it was possible to choose a constant value which resulted in a parking orbit closely approximating the desired parking orbit. This study is an examination of the effectiveness and the efficiency of using a constant value of K as the thrust control parameter.

RESULTS AND DISCUSSION

Comparison of Constant-Angle and Optimum Trajectories

Results obtained by using various constant values of K are shown in figure 5 for thrust initiated at various points along the nominal approach trajectory. Combinations of constant thrust angle K and thrust time which were used for establishing parking orbits are shown in figure 5(a) as a function of thrust-initiation altitude. Shown in figure 5(b) are the pericynthion and apocynthion altitudes of the resulting parking orbits. Table I shows in more detail the initial and terminal conditions and the parking-orbit characteristics established for three of the representative thrust-initiation altitudes of figure 5 for both constant-angle and optimum trajectories. Figure 5(a) shows that K decreases with decreasing thrust-initiation altitude from a value of 38.6° to 12°. A larger value of K is required at the higher altitude because of the larger negative values of radial velocity. (See table I.) Figure 5(a) also indicates that thrust time varies only a few seconds and is essentially independent of the thrust-initiation altitude. The resulting orbits (fig. 5(b)) established by using various combinations of constant K and thrust time are near the desired 80-nautical-mile (148 160 m) circular orbit. The maximum deviation from the desired orbit is 5 nautical miles (9260 m).

A comparison of the thrust time and characteristic velocity required to establish the parking orbits for the optimum and constant-angle transfer trajectories is shown in figure 6. Thrust time and characteristic velocity are shown as a function of thrust-initiation altitude for both fuel-optimum and constant-angle trajectories. The parking orbits for the optimum trajectories are circular 80-nautical-mile altitude orbits whereas

TABLE I.- COMPARISON OF CONSTANT-ANGLE AND OPTIMUM TRAJECTORIES

Parameter	Values of parameters at representative altitudes of -					
	721 000 ft (219 761 m)		586 000 ft (178 613 m)		500 000 ft (152 400 m)	
	Optimum trajectory	Constant-angle trajectory, $K = 38.6^{\circ}$	Optimum trajectory	Constant-angle trajectory, $K = 25.8^{\circ}$	Optimum trajectory	Constant-angle trajectory, $K = 12.0^{\circ}$
Initial conditions						
Altitude, h	720 720 ft (219 675 m)	720 720 ft (219 675 m)	586 330 ft (178 713 m)	586 330 ft (178 713 m)	500 148 ft (152 445 m)	500 148 ft (152 445 m)
Radial velocity, $\dot{r}$	-1709.29 ft/sec (-520.99 m/s)	-1709.29 ft/sec (-520.99 m/s)	-985.09 ft/sec (-300.25 m/s)	-985.09 ft/sec (-300.25 m/s)	-447.41 ft/sec (-136.37 m/s)	-447.41 ft/sec (-136.37 m/s)
Transverse velocity, $r\dot{\theta}$	8 003.36 ft/sec (2439.42 m/s)	8 033.36 ft/sec (2448.57 m/s)	8209.13 ft/sec (2502.14 m/s)	8209.13 ft/sec (2502.14 m/s)	8287.99 ft/sec (2526.18 m/s)	8287.99 ft/sec (2526.18 m/s)
Terminal conditions						
Altitude, h	486 089 ft (148 160 m)	487 751 ft (148 667 m)	486 089 ft (148 160 m)	454 675 ft (138 585 m)	486 089 ft (148 160 m)	488 803 ft (148 987 m)
Radial velocity, $\dot{r}$	0 ft/sec (0 m/s)	-1.43 ft/sec (-0.44 m/s)	0 ft/sec (0 m/s)	-3.39 ft/sec (-1.03 m/s)	0 ft/sec (0 m/s)	-2.50 ft/sec (-0.76 m/s)
Transverse velocity, $r\dot{\theta}$	5286.90 ft/sec (1611.45 m/s)	5284.40 ft/sec (1610.69 m/s)	5286.90 ft/sec (1611.45 m/s)	5307.60 ft/sec (1617.76 m/s)	5286.90 ft/sec 1611.45 m/s)	5286.40 ft/sec (1611.29 m/s)
Angular travel, θ	19.5°	19.5°	19.6°	19.6°	20.0°	20.0°
Thrust time, t	315 sec	315 sec	310 sec	310 sec	314 sec	314 sec
Parking orbits						
Minimum altitude, h_P	80.0 n. mi. (148 160.0 m)	78.8 n. mi. (145 937.6 m)	80.0 n. mi. (148 160.0 m)	74.8 n. mi. (138 529.6 m)	80.0 n. mi. (148 160.0 m)	80.2 n. mi. (148 530.4 m)
Maximum altitude, h_A	80.0 n. mi. (148 160.0 m)	80.3 n. mi. (148 715.6 m)	80.0 n. mi. (148 160.0 m)	80.4 n. mi. (148 900.8 m)	80.0 n. mi. (148 160.0 m)	81.2 n. mi. (150 382.4 m)

the constant-angle parking orbits are characterized by the apocynthion and pericynthion altitudes shown in figure 5. Figure 6 shows that the use of a constant angle is an efficient procedure and figure 5 shows that the desired orbit characteristics can be attained.

Error Analysis

A constant-angle braking trajectory with initial conditions corresponding to those of the nominal approach trajectory at an altitude of 720 720 feet (219 675 m) was arbitrarily chosen as the nominal braking maneuver for the purpose of an error analysis. The initial conditions, terminal conditions, and parking orbit associated with this trajectory are listed in table I.

Individual errors.- The effects of single errors in thrust-vector orientation, thrust level, thrust time, initial altitude, and initial velocity on the nominal parking orbit were obtained by varying only one of these conditions from its nominal value at a time, computing the braking trajectory, and then noting the variations of the resulting parking orbit from the nominal orbit. These variations are shown in figure 7 in terms of apocynthion and pericynthion altitudes. The error ranges presented are much larger than the anticipated error ranges likely to occur.

The results presented in figure 7 indicate several points of interest: (a) The parking orbit resulting from a single error has a pericynthion altitude less than or approximately equal to the nominal value $\left(h_P \lesssim \text{Nominal } h_P\right)$ and an apocynthion altitude greater than or approximately equal to the nominal value $\left(h_A \gtrsim \text{Nominal } h_A\right)$ and (b) the effects of positive errors or negative errors on the apocynthion and pericynthion altitudes are approximately linear. The slopes of the curves for Δh_P and Δh_A with positive errors are almost the same as the slopes of the curves for Δh_A and Δh_P with negative errors, respectively.

The following table is a list of the effects on h_P and h_A of some errors having magnitudes believed to be conservative estimates of those that might occur in a normal mission phase:

Parameter	Error in parameter	Δh_P	Δh_A
ΔK	1.0°	-5.0 n. mi. (-9 260.0 m)	13.3 n. mi. (24 631.6 m)
	-1.0°	-13.3 n. mi. (-24 631.6 m)	6.7 n. mi. (12 408.4 m)
$\Delta F/W_o$	1.0 percent	-26.7 n. mi. (-49 448.4 m)	--------------------
	-1.0 percent	----------------------	26.7 n. mi. (49 448.4 m)
Δt	1.0 sec	-8.3 n. mi. (-15 371.6 m)	--------------------
	-1.0 sec	----------------------	8.3 n. mi. (15 371.6 m)
Δh_o	2.0 n. mi. (3704.0 m)	0.4 n. mi. (740.8 m)	5.3 n. mi. (9 815.6 m)
	-2.0 n. mi. (-3704.0 m)	-5.0 n. mi. (-9 260.0 m)	--------------------
$\Delta \dot{r}_o$	10.0 ft/sec (3.048 m/s)	-2.0 n. mi. (-3 704.0 m)	1.2 n. mi. (2 222.4 m)
	-10.0 ft/sec (-3.048 m/s)	1.1 n. mi. (2 037.2 m)	2.1 n. mi. (3 889.2 m)
$\Delta (r\dot{\theta})_o$	10.0 ft/sec (3.048 m/s)	----------------------	7.7 n. mi. (14 260.4 m)
	-10.0 ft/sec (-3.048 m/s)	-7.7 n. mi. (-14 260.4 m)	--------------------

Although lunar-surface irregularities will produce an error in K; this error does not appear serious. For example, a 5000-foot-peak (1524 m) mountain will only produce a maximum error in K of about 0.1°. Thus, from a consideration of the errors shown in the preceding table, it appears that safe parking orbits can be obtained under a wide range of errors.

Correction of errors.- If the spacecraft is on an approach trajectory which varies slightly from the nominal trajectory, it will have different velocity components when it reaches the nominal altitude, or time, at which the braking maneuver is to be initiated. If these off-nominal values of velocity can be accurately determined, appropriate corrections can be made in the thrust angle and thrust time to compensate for these off-nominal conditions. Figure 8 shows the appropriate changes in thrust angle and thrust time for various combinations of errors in the velocity components $\dot{r}_o$ and $(r\dot{\theta})_o$ at the nominal altitude $(\Delta h_o = 0)$. Combination errors are considered over a range of ±100 ft/sec

(±30.5 m/s) in each velocity component. The parking orbits resulting from the use of figure 8 are confined to an altitude of approximately 80 ± 5 n. mi. (148 160 ± 9260 m). If no corrections are performed to compensate for the initial-velocity errors and the nominal flight program is used, changes in the parking orbits will result as shown in figure 9. Thus, the corrective action is quite effective. The off-nominal hyperbolic approach trajectories corresponding to the initial-velocity errors shown in figure 8 have pericynthion altitudes and pericynthion velocities which vary about ±5 n. mi. and ±50 ft/sec (±15.2 m/s), respectively, from the nominal values.

The corrective action in K and t indicated by figure 8 can also be represented approximately by the following two linear equations:

$$\Delta K = C_1 \Delta \dot{r}_o + C_2 \Delta (r\dot{\theta})_o \tag{4}$$

$$\Delta t = C_3 \Delta \dot{r}_o + C_4 \Delta (r\dot{\theta})_o \tag{5}$$

where the constants (partial derivatives) are obtained from the figure and are given by

$$C_1 = -0.012 \frac{\text{deg}}{\text{ft/sec}} = -0.039 \frac{\text{deg}}{\text{m/sec}}$$

$$C_2 = -0.017 \frac{\text{deg}}{\text{ft/sec}} = -0.056 \frac{\text{deg}}{\text{m/sec}}$$

$$C_3 = -0.03 \frac{\text{sec}}{\text{ft/sec}} = -0.10 \frac{\text{sec}}{\text{m/sec}}$$

$$C_4 = 0.07 \frac{\text{sec}}{\text{ft/sec}} = 0.23 \frac{\text{sec}}{\text{m/sec}}$$

Application of Simplified Guidance Technique

A limited simulation study (ref. 6) has indicated that the technique developed in this paper may be successfully used by the pilot in controlling the establishment of a lunar parking orbit. Also, there is no reason to believe that the application of the simplified technique is limited only to the moon; it might also be applied to missions involving other planetary bodies of the solar system.

CONCLUDING REMARKS

An analytical study has been made of a simplified guidance technique for establishing a lunar parking orbit from a hyperbolic approach trajectory. The guidance technique consists of orienting the thrust-vector attitude at a constant angle with respect to the receding lunar horizon during the retrothrust maneuver. Results of the investigation indicated that a lunar orbit can be established which varies, at most, 5 nautical miles (9260 m) from an 80-nautical-mile (148 160 m) circular orbit (the target orbit) for braking maneuvers initiated over a range of altitudes along the hyperbolic approach trajectory. In addition, a comparison of these constant-angle maneuvers with those of fuel-optimum maneuvers indicated that the constant-angle maneuvers were relatively efficient. An error analysis was made which showed that safe parking orbits could be established by using conservative estimates of the magnitude of various error sources when these errors were considered singly. No attempt was made to make a complete error analysis. Hyperbolic approach trajectories which differed from the nominal trajectory, but which passed through the nominal altitude of 720 720 feet (219 675 m), were investigated. A scheme was devised to vary the magnitude of the constant angle with respect to the lunar horizon and the thrust time to account for the different hyperbolic approach trajectories. This scheme resulted in errors of not more than ±5 nautical miles in the desired 80-nautical-mile parking orbit.

A limited simulation study has indicated that the technique developed in this paper may be successfully used by the pilot in controlling the establishment of a lunar parking orbit. Also, there is no reason to believe that the application of the simplified technique is limited only to the moon; it might also be applied to missions involving other planetary bodies of the solar system.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., December 10, 1965.

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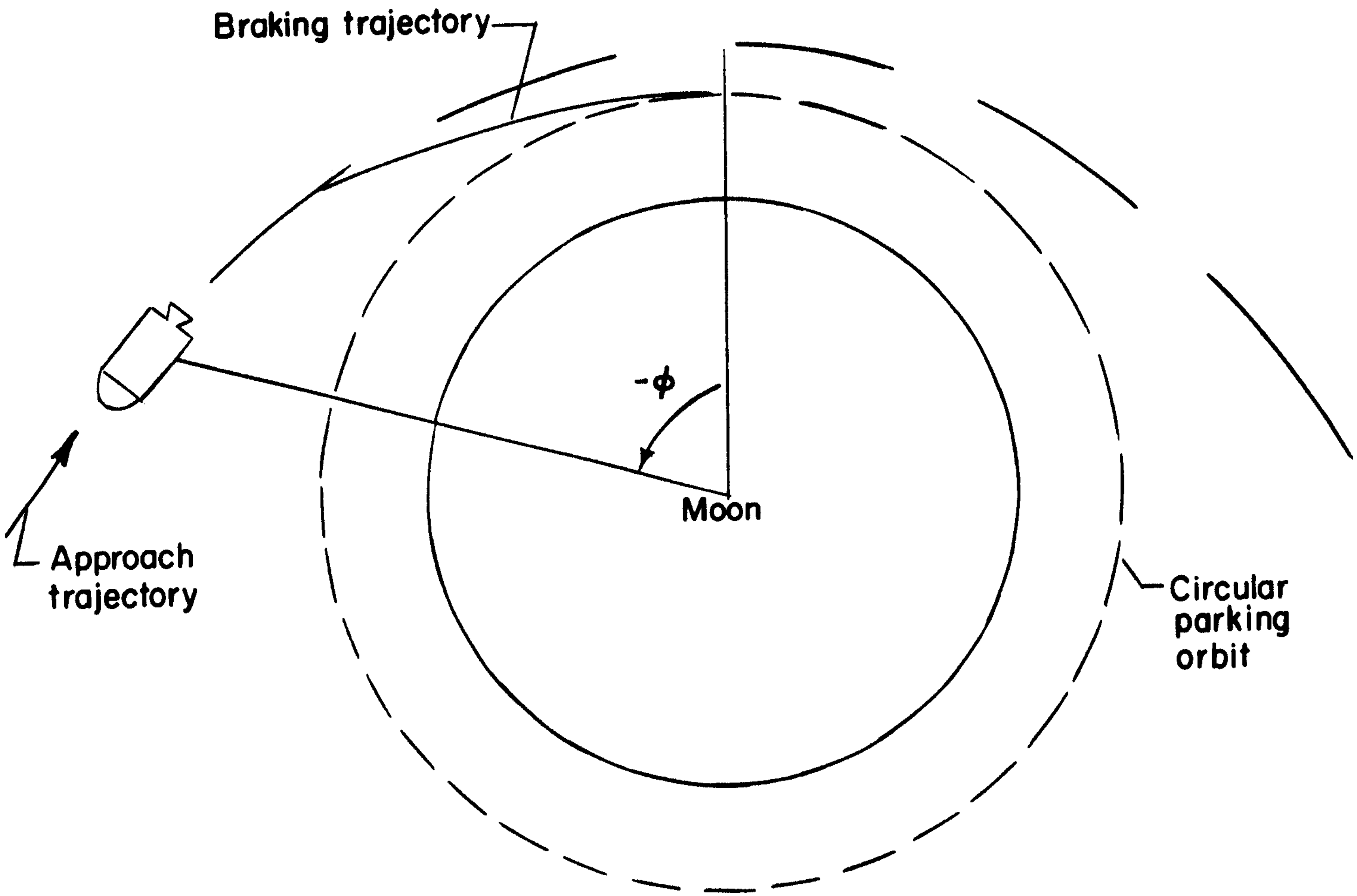


Figure 1.- Illustration of mission phase.

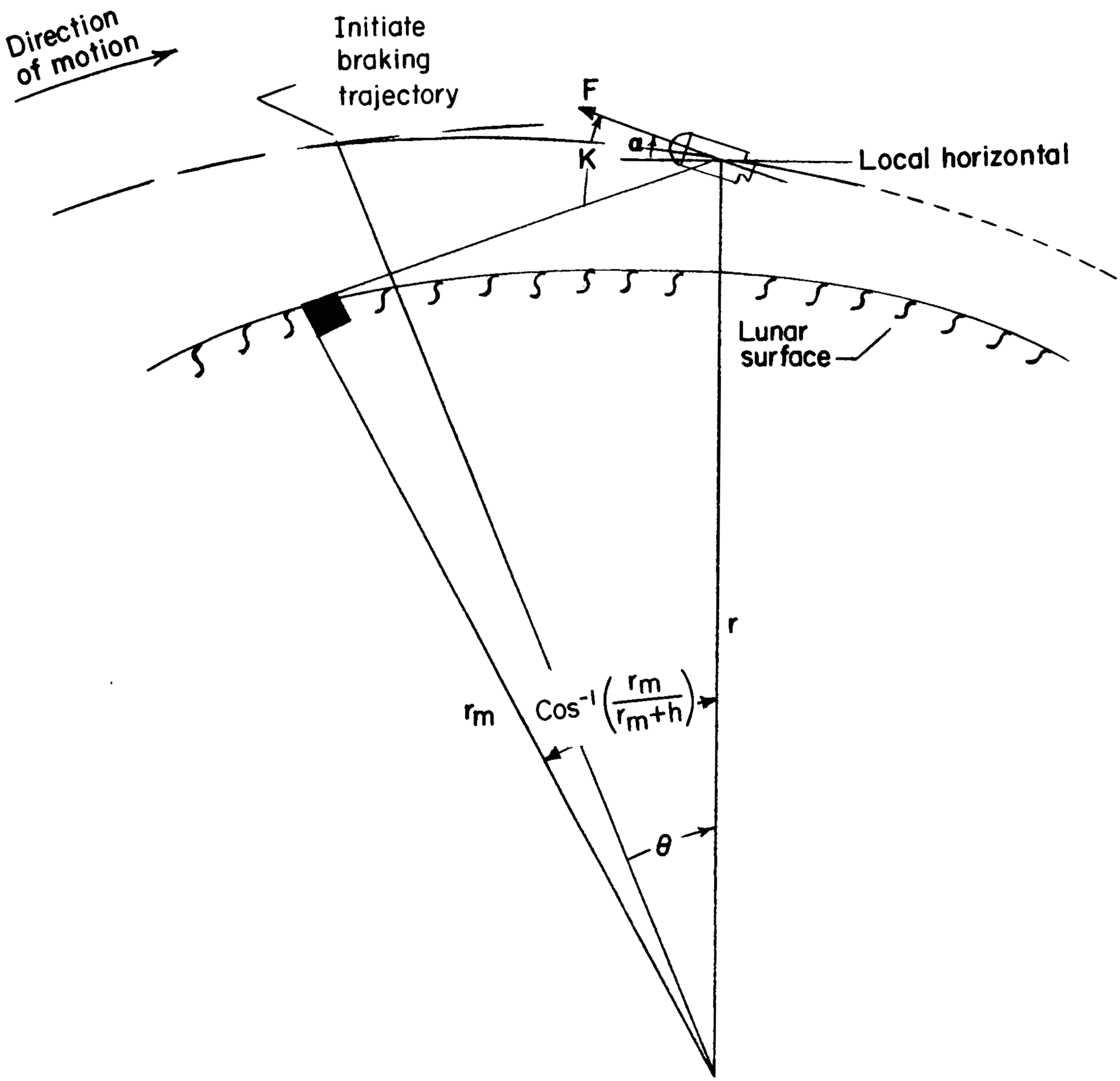


Figure 2.- Geometry of braking trajectory.

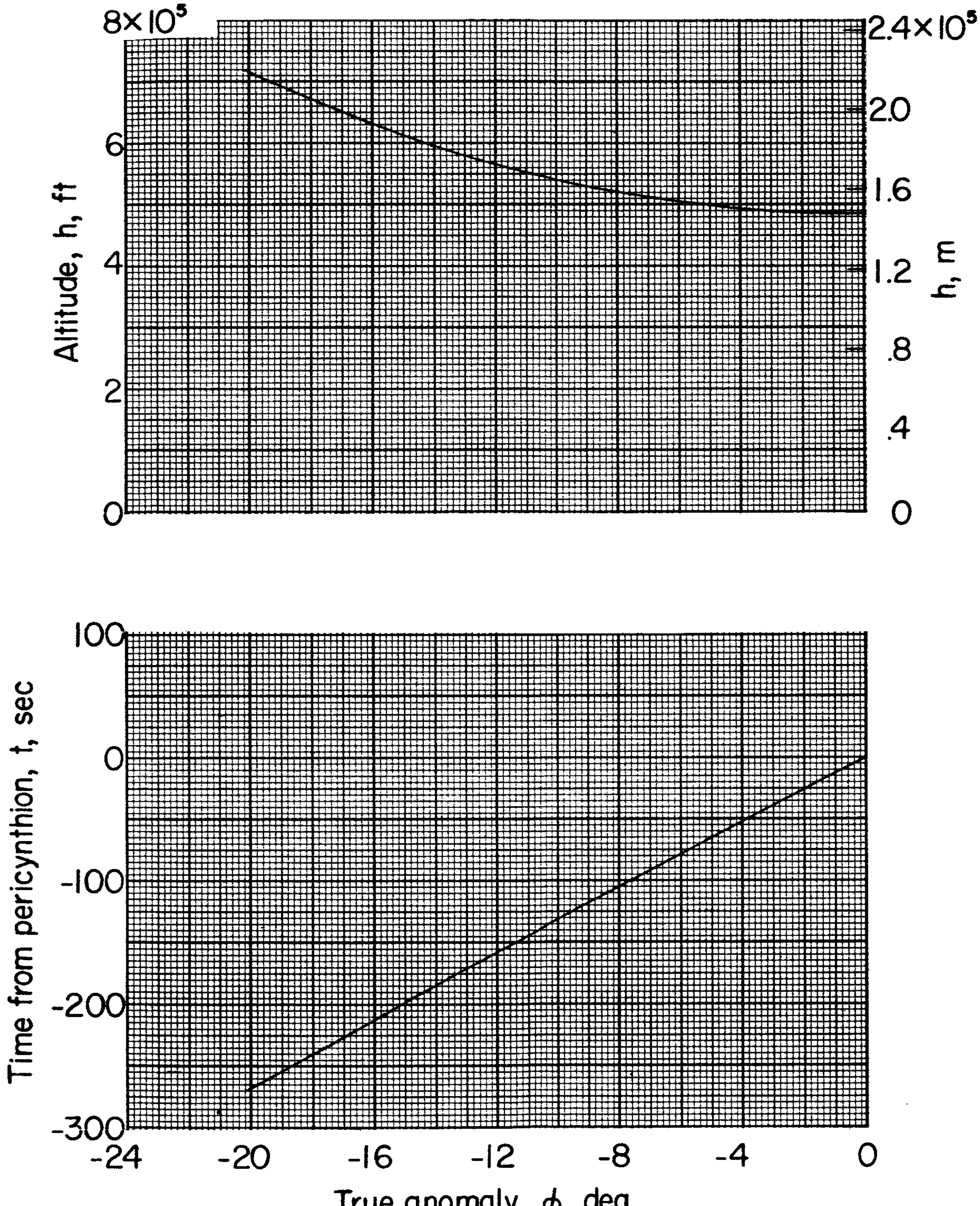


Figure 3.- True-anomaly history of nominal approach trajectory.

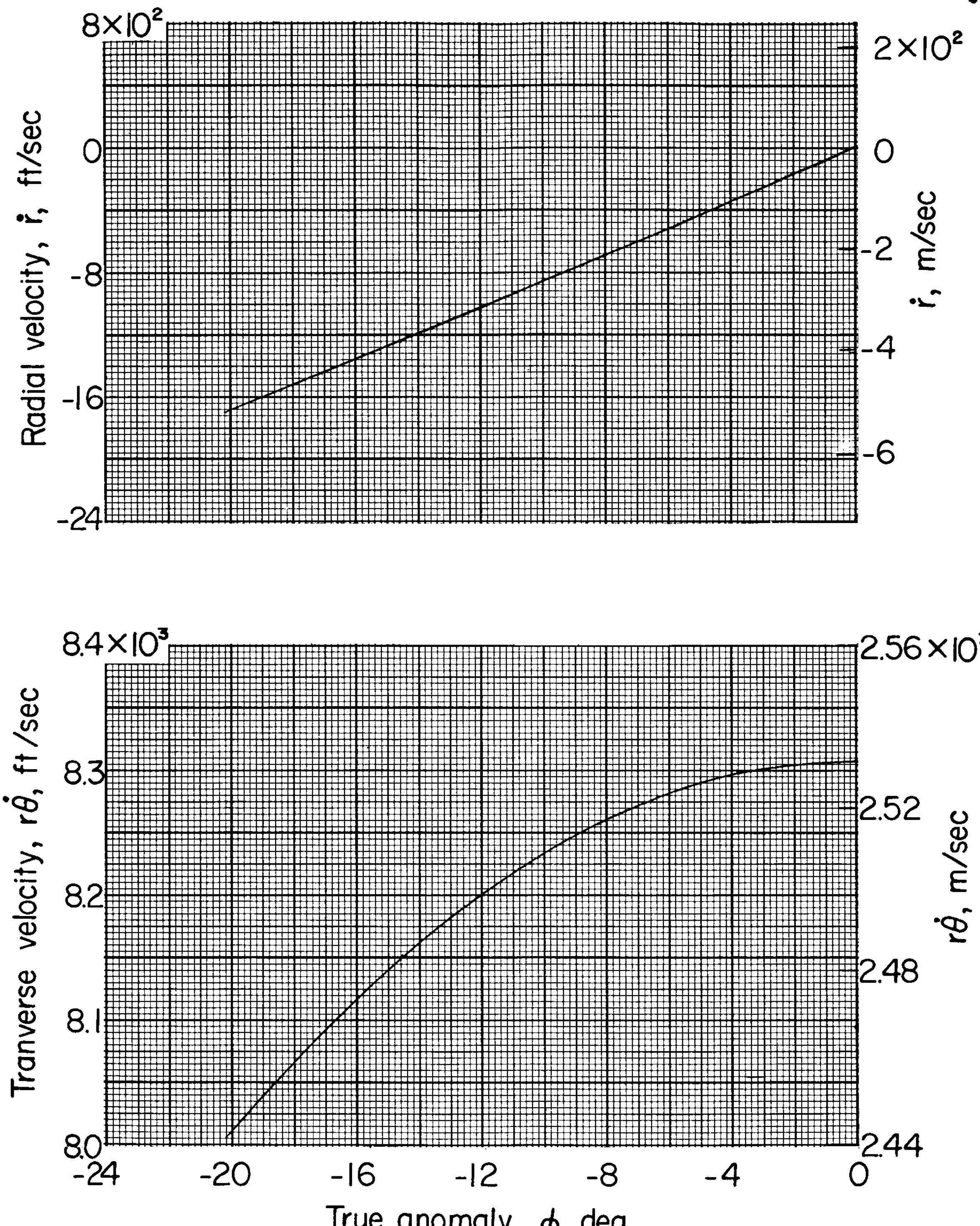


Figure 3.- Concluded.

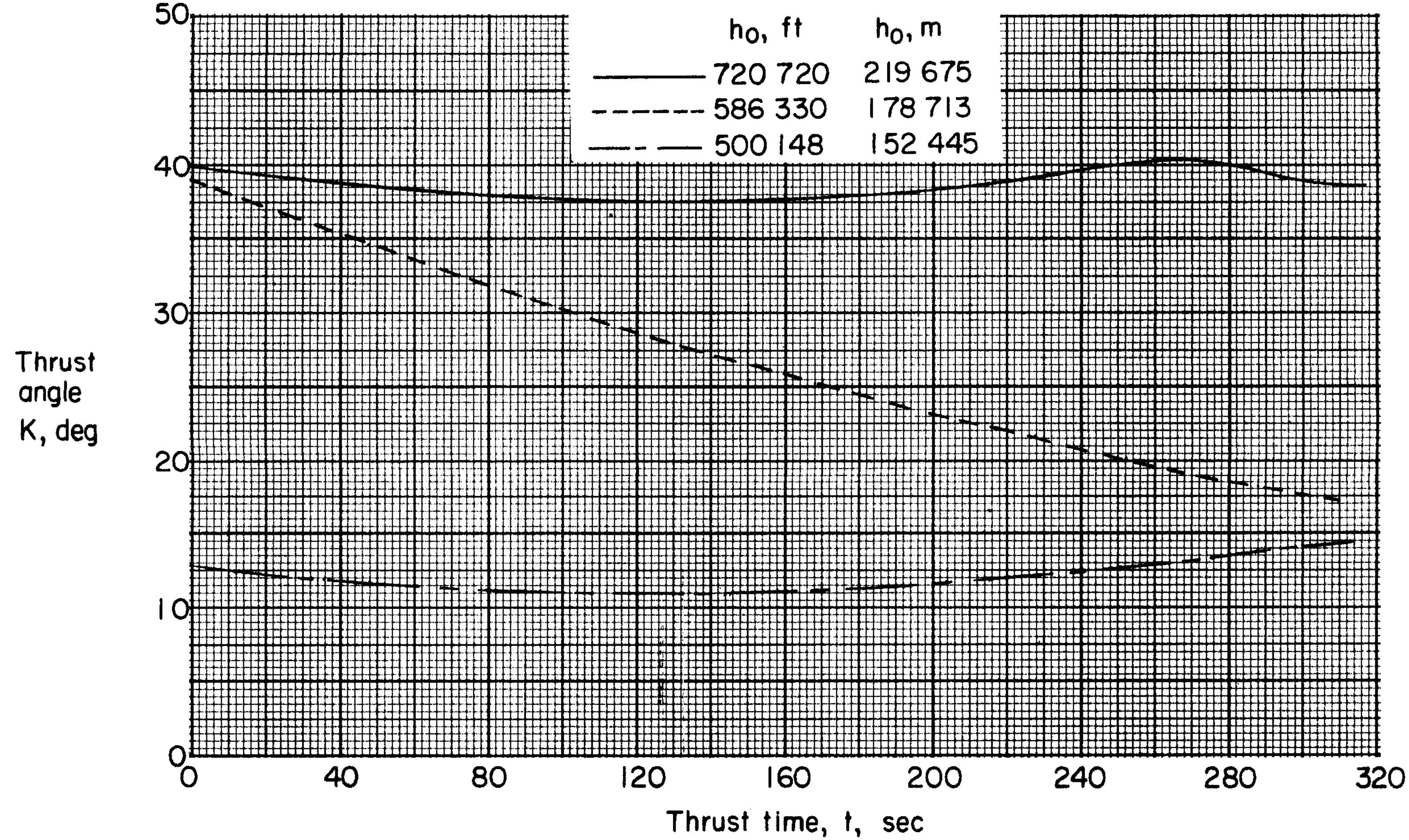


Figure 4.- Variation of angle between thrust vector and line of sight to lunar horizon for fuel-optimum braking trajectories.

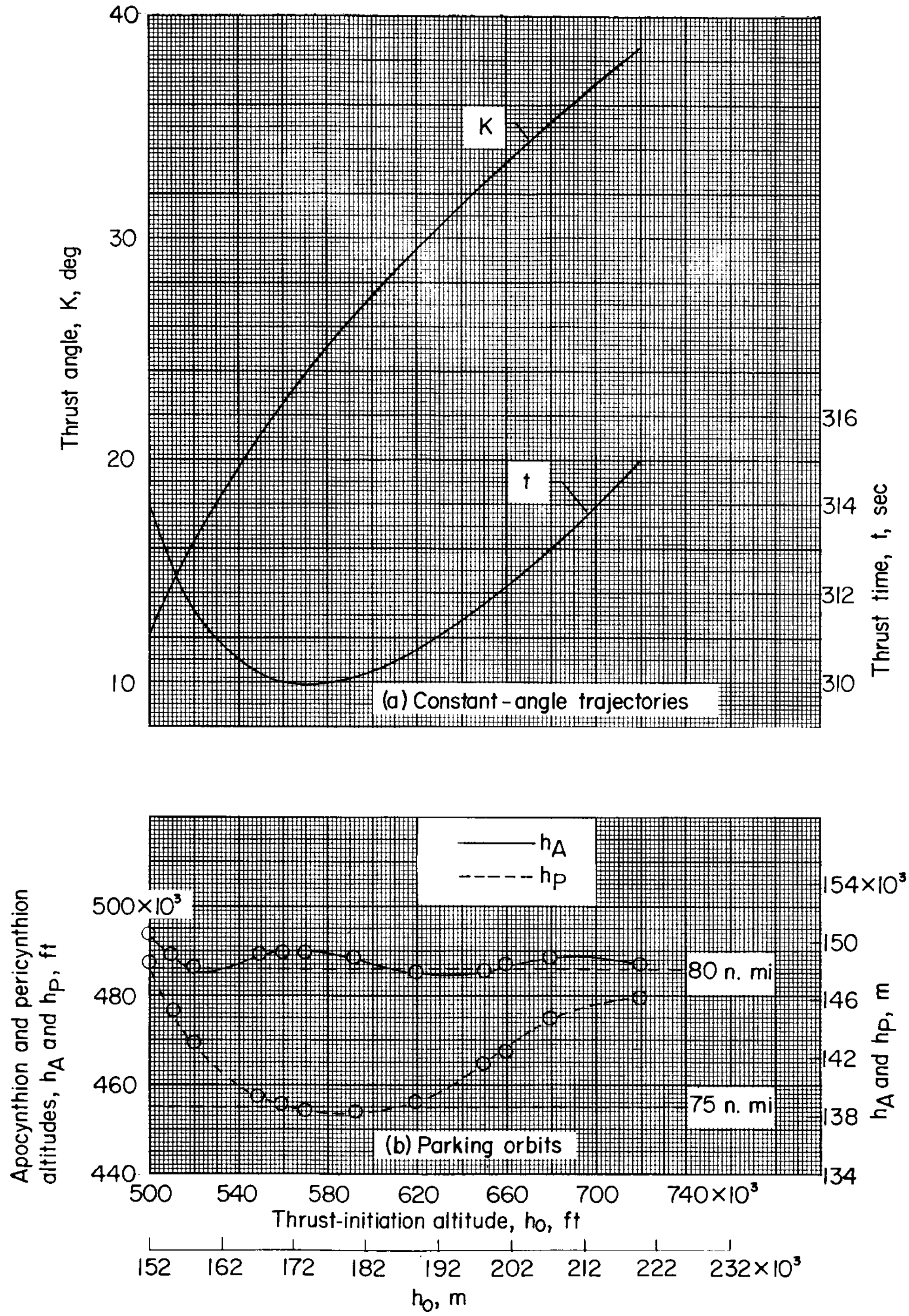


Figure 5.- Constant-angle trajectories and subsequent parking orbits for various thrust-initiation altitudes.

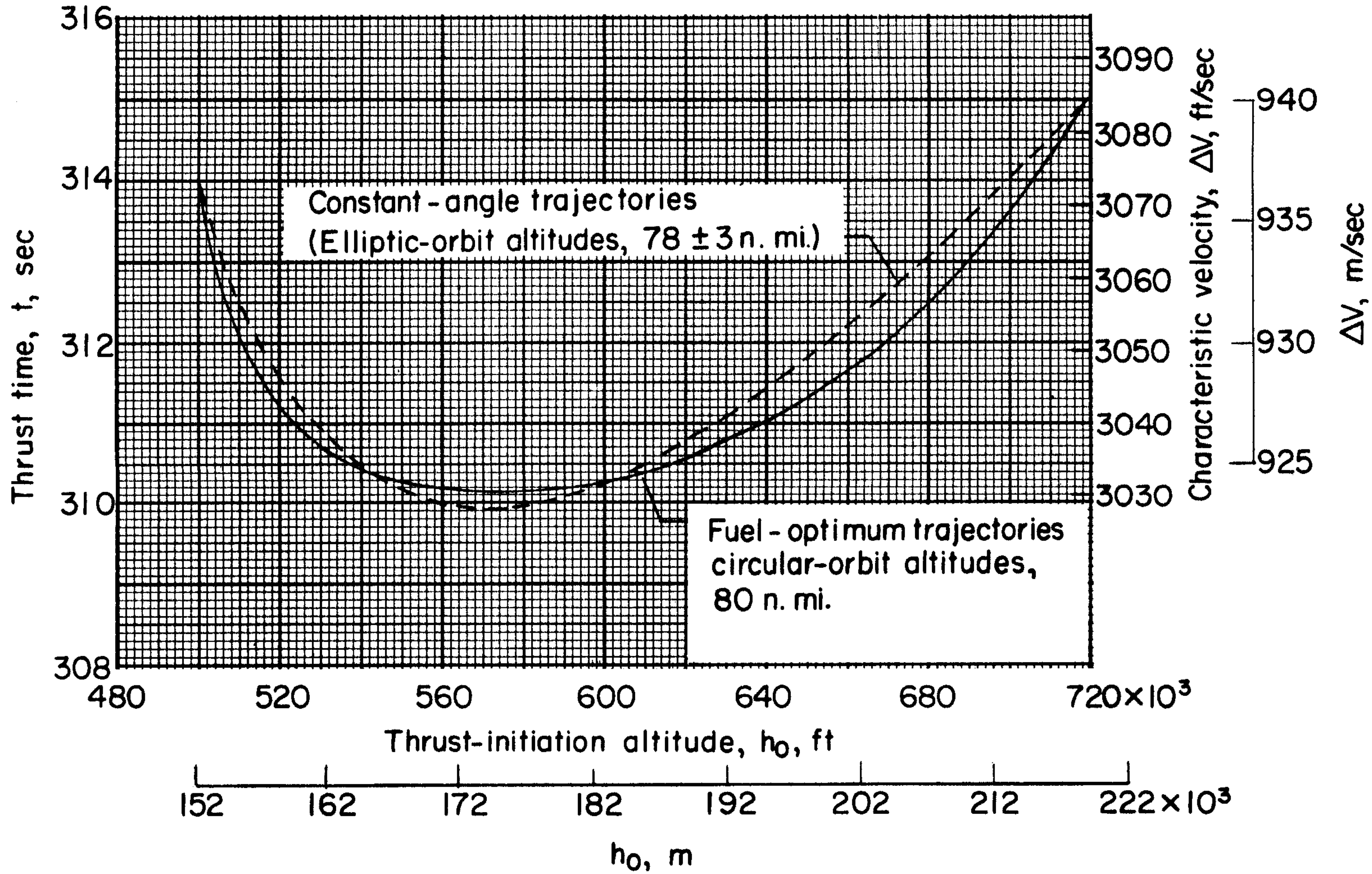
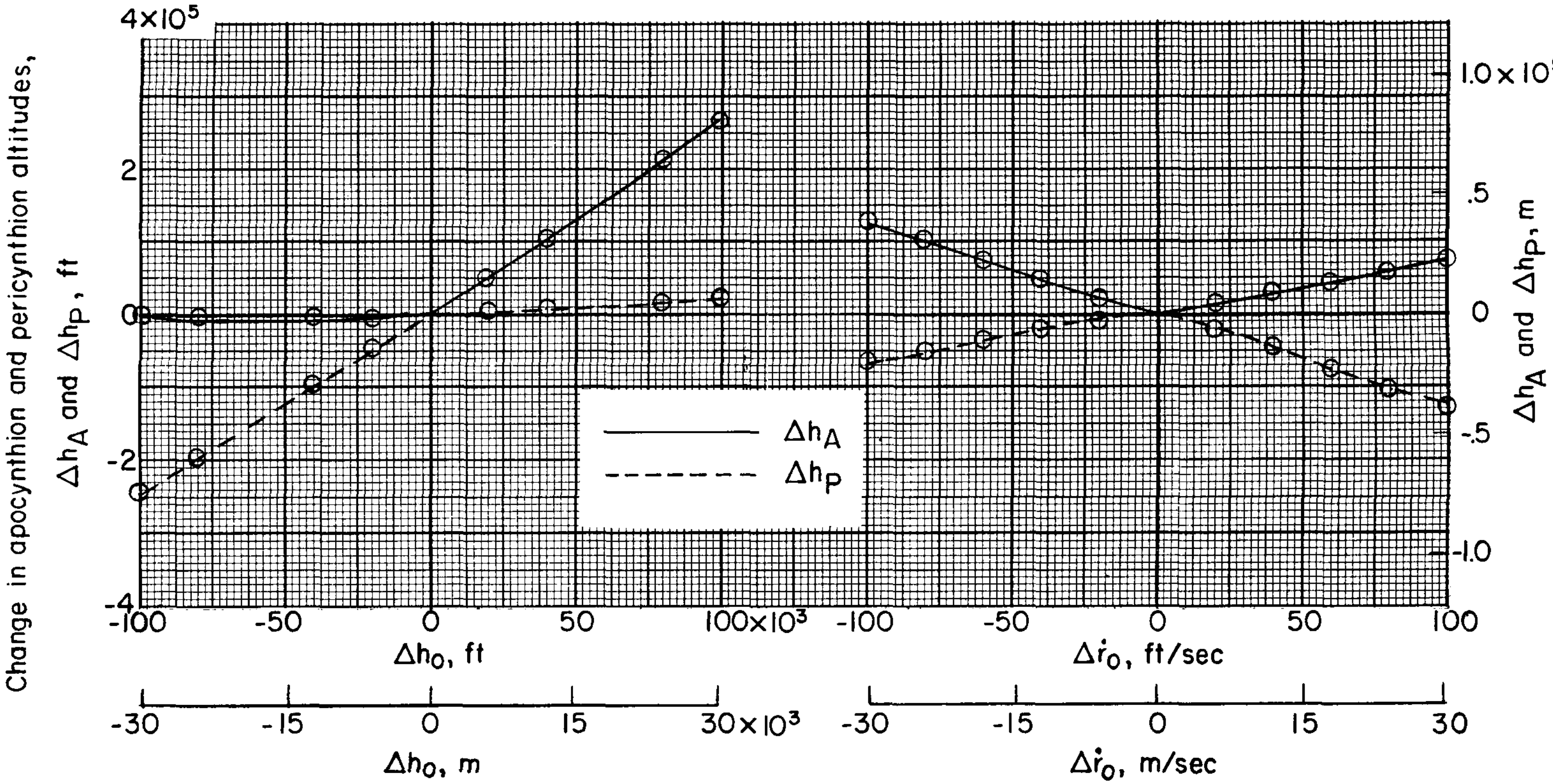


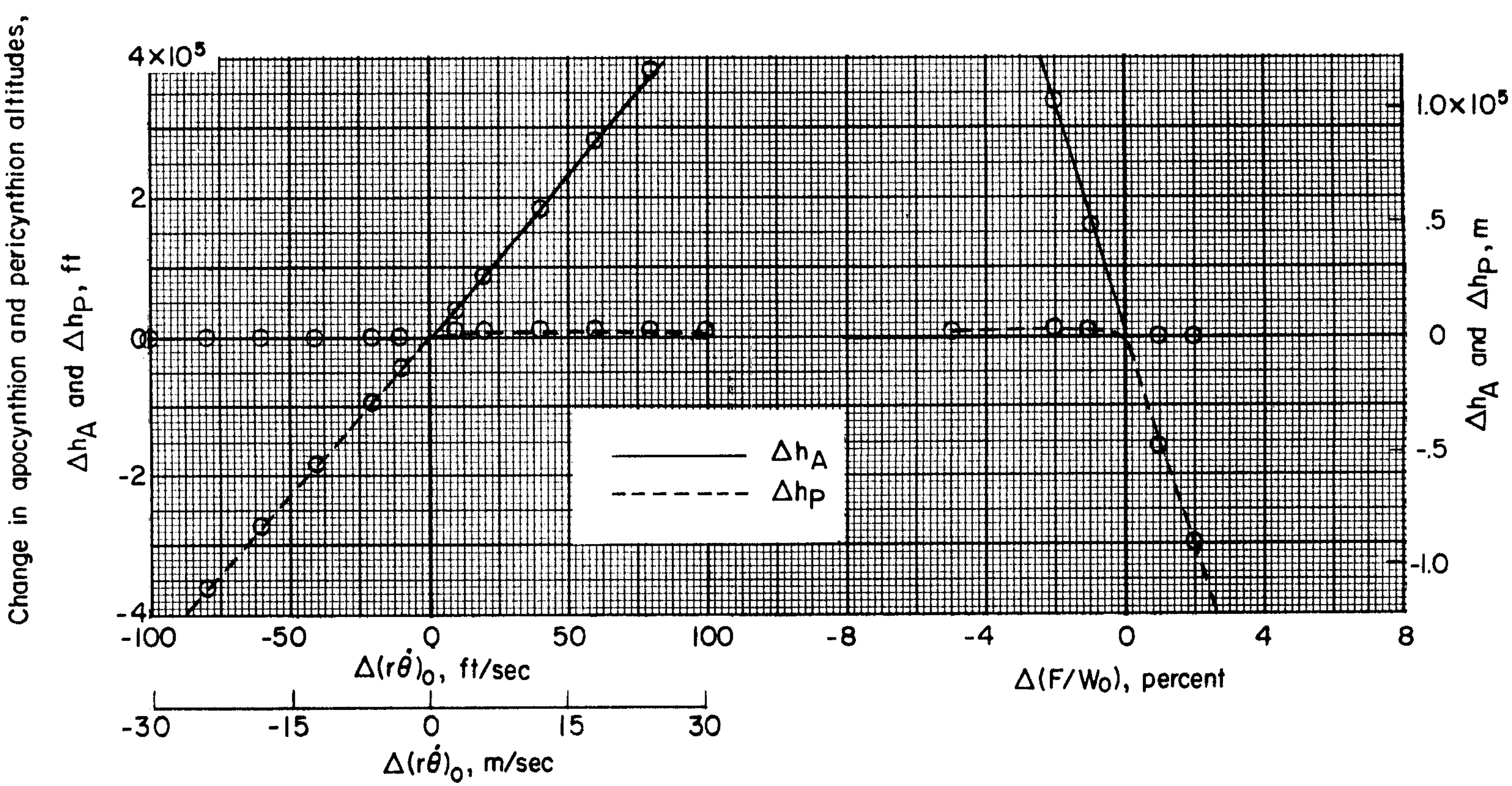
Figure 6.- Thrust-time and characteristic velocity requirements of fuel-optimum and constant-angle braking trajectories for different thrust-initiation altitudes.



(a) Thrust-initiation-altitude error.

(b) Radial-velocity error.

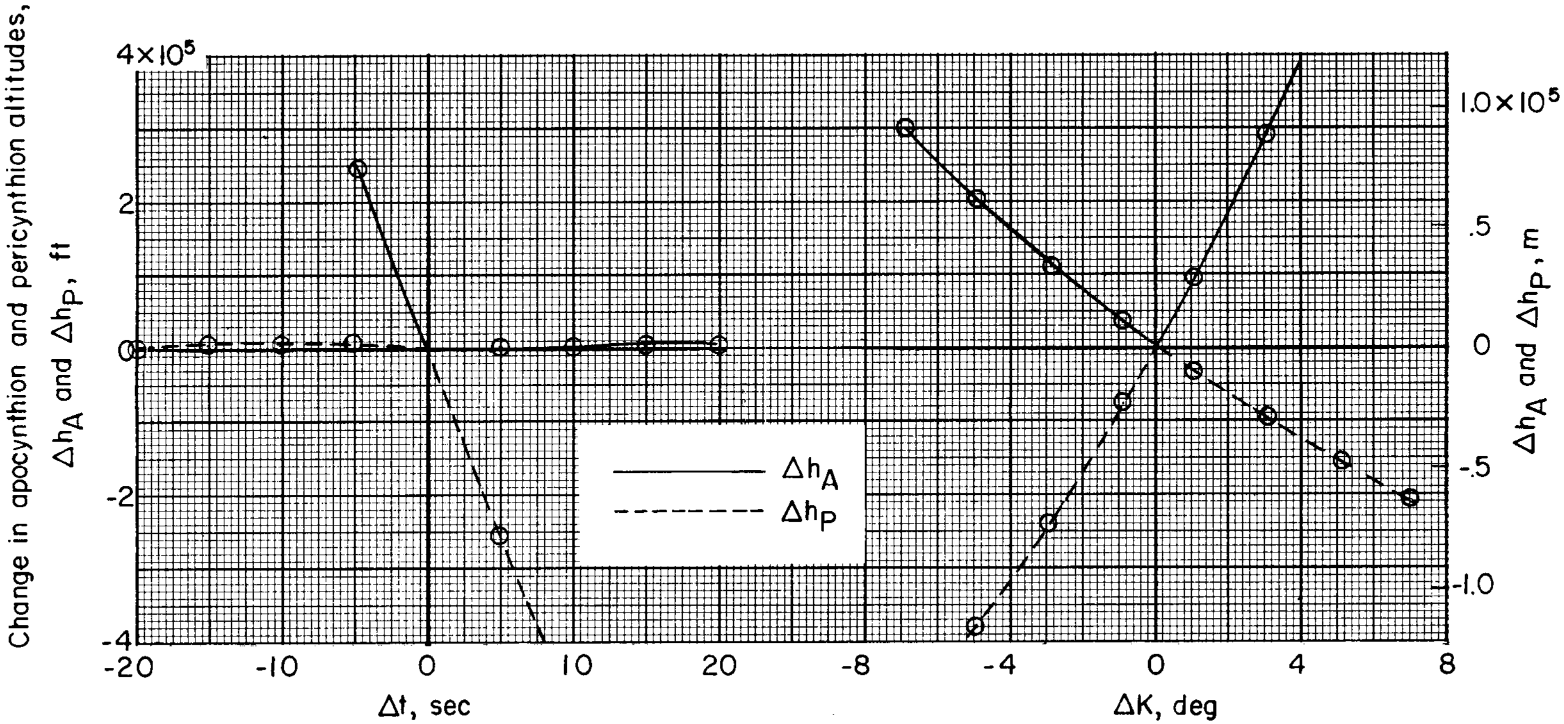
Figure 7.- Effects of errors on the nominal parking orbit.



(c) Transverse-velocity error.

(d) Thrust-weight-ratio error.

Figure 7.- Continued.



(e) Thrust-time error.

(f) Thrust-angle error.

Figure 7.- Concluded.

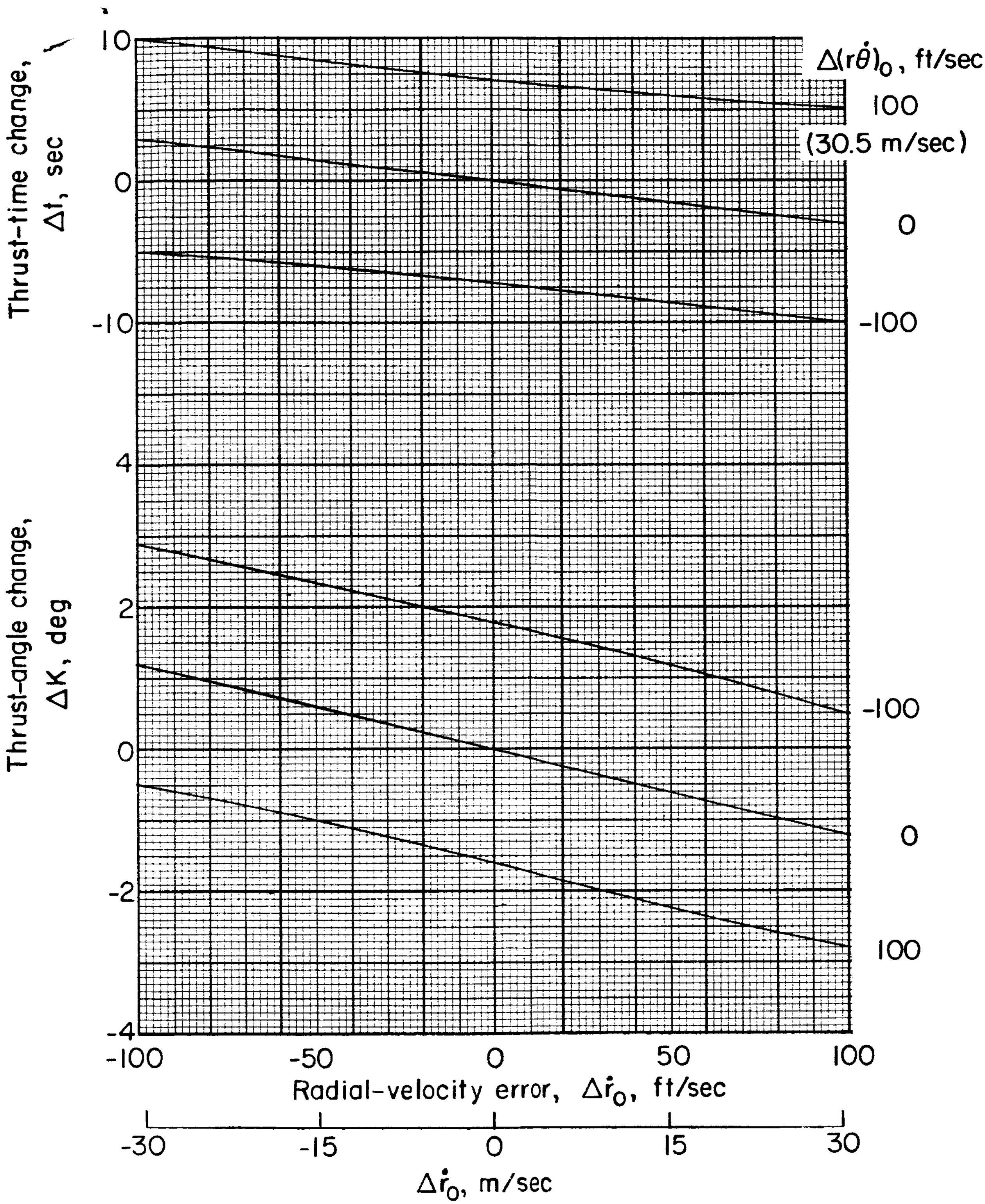


Figure 8.- Correction method for initial velocity errors. (Resulting parking orbit altitudes of 80 ± 5 n. mi.)

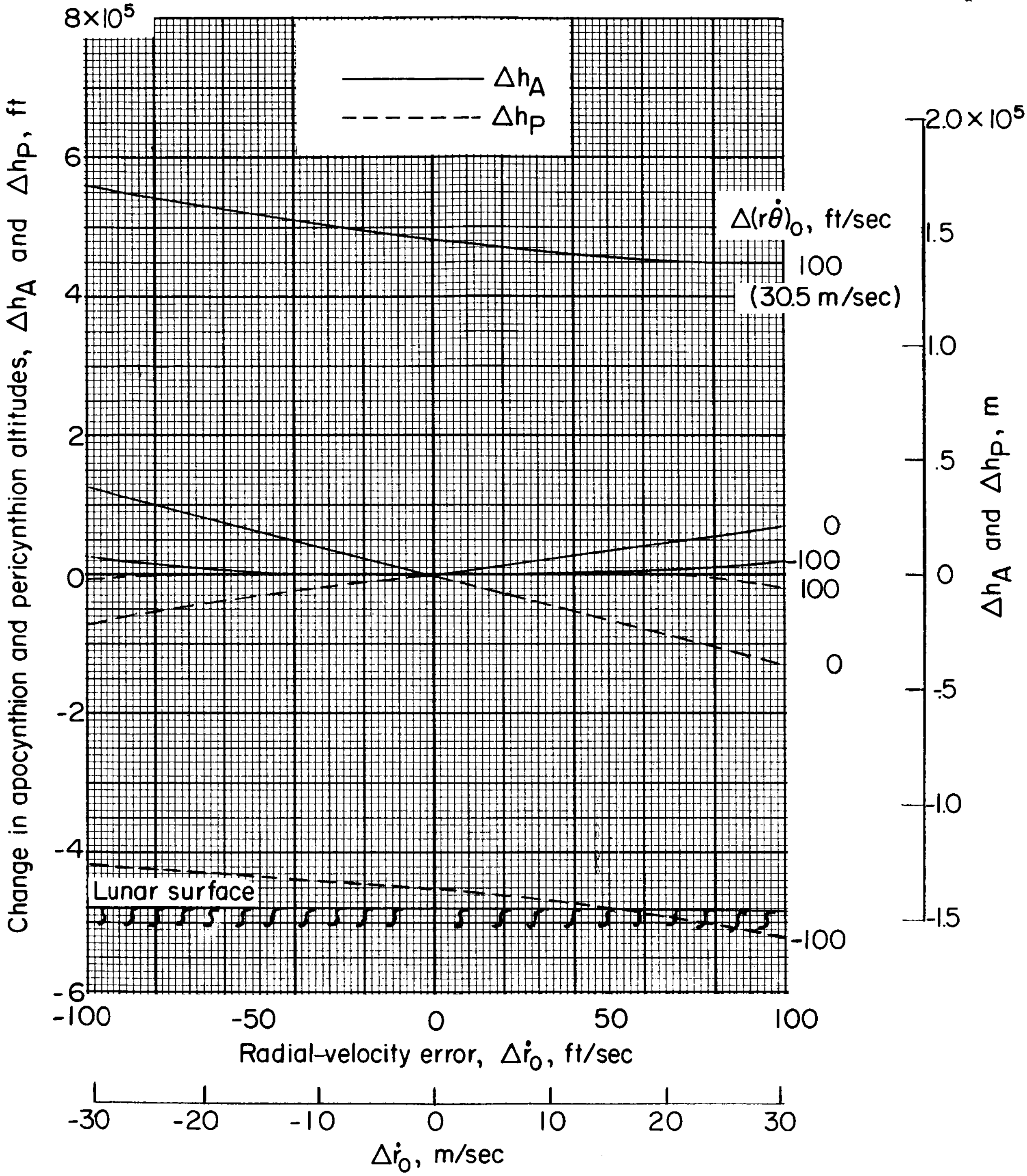


Figure 9.- Effect of initial velocity errors on the nominal parking orbit.

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